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ON THE NATURE OF QUASI-PERIODIC OSCILLATIONS IN THE TAIL OF SOFT GAMMA REPEATER GIANT FLARES

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ABSTRACT

A model is presented for the quasi-periodic component of magnetar emission during the tail phase of giant flares. The model invokes modulation of the particle number density in the magnetosphere. The magnetospheric currents are modulated by torsional motion of the surface, and we calculate that the amplitude of neutron star (NS) surface oscillation should be $\sim$1% of the NS radius in order to produce the observed features in the power spectrum. Using an axisymmetric analytical model for structure of the magnetosphere of an oscillating NS, we calculate the angular distribution of the optical depth to the resonant Compton scattering. The anisotropy of the optical depth may be why quasi-periodic oscillations are observed only at particular rotational phases.

Subject headings: stars: magnetic fields — stars: neutron — stars: oscillations

1. INTRODUCTION

Duncan (1998) proposed that a magnetar undergoing a giant flare would be set into oscillations. Timokhin et al. (2000) and Timokhin (2007) proposed that torsional oscillations could modulate the local Goldreich-Julian charge density and noted that changes in the Goldreich-Julian charge density are proportional to the harmonic numbers l and m. Israel et al. (2005) and Strohmayer & Watts (2005) have reported detections of quasi-periodical features in the power spectra of SGR 1806−20 and 1900+40, which they interpret to be $n = 0$ high-l torsional oscillations of the crust. These features in the power spectra were detected mostly in hard (probably nonthermal) X-rays in the energy range up to 200 keV; the frequencies of these quasi-periodic oscillations (QPOs) range from 18 Hz up to 600 Hz. The QPO emission is observed only during certain rotational phases. One of several remarkable features of this discovery is the rather large amplitude of the luminosity modulation, of the order of 10% for the high-l modes, given that a crust would break or melt if the crustal oscillations were too violent. The shear strain of crustal oscillations is probably limited to about $\sim$10%, which for $l \sim 10$, would result in a limit for the oscillation amplitude about one or two percent of the neutron star (NS) radius $R_{\rm NS}$, unless vortex pinning and/or strong magnetic field make the crust more resilient.

Even more remarkable is that such large modulation appears most prominently at certain high-l harmonics. Even if temperature fluctuations of order several percent could be somehow achieved at the photosphere, it would not lead to such large luminosity modulation if many maxima and minima were visible to the observer at any given time. Moreover, the high value of l suggests that energy in crustal oscillations is triggered from a relatively small part of the crust's surface and is distributed into a large number of modes. It also raises the possibility that the deep crustal heating that can account for the medium- and long-term X-ray afterglow following giant flares and other periods of magnetic activity (Lyubarsky et al. 2002; Kouveliotou et al. 2003) may be powered by crustal oscillations.

In this paper we propose a model for the amplitude variations that appears to support the idea that the oscillations are indeed torsional oscillations of the crust and may also support the suggestions (Thompson et al. 2002) that the magnetospheric currents play a significant role in the magnetar emission at high photon energy.

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2. THE MODEL

We suggest that the modulation of the hard X-ray radiation visible in the tail of the giant flares is caused by modulation of the particle number density in the magnetosphere due to shaking of the magnetic field lines by oscillations of the NS crust. The specific candidate emission mechanisms can be cyclotron scattering of thermal photons (Lyutikov & Gavriil 2006), nonresonant scattering of thermal photons, or bremsstrahlung off a target corona, although a detailed investigation of the latter two is not attempted here, as they seem to us less promising. The electrical resistance created by photon drag during the tail phase of a giant flare is several orders of magnitude greater—as is the non-QPO photon background against which the QPO component must stand out—than during the persistent emission. This consideration leads us to favor resonant scattering, for which the optical depth offered by the current carriers to the thermal photons is the greatest.[2]

In order to support the twist of the magnetic field, a current density j must flow along magnetic field lines. The minimum particle density in the magnetosphere of the magnetar is therefore $j/(ec)$. The strength of the magnetic field where resonant inverse Compton scattering on electrons occurs is, in the absence of Doppler shift,

$$B_{\rm res} = 0.086 E_{\rm ph}^{[\rm keV]} 10^{12}\ {\rm G}, \tag{1}$$

where $E_{\rm ph}^{[\rm keV]}$ is the initial energy of the scattered photons in units of keV. We call the surface in the magnetosphere where resonant inverse Compton scattering on electrons occurs the "resonant surface." For radially streaming photons and dipole magnetic field, the position of this surface is given by the expression

$$r_{\rm res}(\theta) = R_{\rm NS}\left(\frac{B_0}{B_{\rm res}}\right)^{1/3}\left(1 - \frac{3}{4}\sin^2\theta\right)^{1/6}, \tag{2}$$

[2] The coronal density may itself be enhanced during tail-phase emission by radiative trapping of plasma on closed field lines, and this might also be a source of enhanced electrical resistivity that directly taps the energy in magnetic field twist. However, here we focus only on resonant scattering.

where R_{NS} is the NS radius and B_0 is the magnetic field strength at the pole. For magnetars with the surface magnetic field 10^{15}–10^{14} G, the resonant surface for hard X-rays lies at the distance of the order of $\sim$10 stellar radii (R_{NS}) or less. For the observed frequencies of the QPOs, the electromagnetic wavelengths $\lambda_{QPO} = c/\nu_{QPO}$ corresponding to these oscillations are of the order $50R_{NS}$–$1700R_{NS}$. Hence, the resonant surface is well within the wave zone, and the magnetosphere can be considered as quasi-static. All physical quantities change with time as $e^{-i\omega t}$.

If the reconfiguration of the magnetic field induces oscillations of the NS crust (Bisnovatyi-Kogan 1995; Duncan 1998), the motion of the star surface moves the magnetic field lines frozen into the surface. The motion of the field lines induces the electric field and, for modes which move the footpoints of a given magnetic field line in opposite directions, induces a twist in the field line. The twist of the field lines induces electric current along the lines and, hence, changes the particle number density in the magnetosphere. As we show in Appendix A, only modes which move the footpoints of a given magnetic field line in opposite directions and, thus, induce the electric current along magnetic field lines could modulate the hard X-radiation of magnetars.

The optical depth to the resonant Compton scattering on a particle with the charge Ze is (cf. eq. [23] in Thompson et al. 2002)

$$\tau(\theta) = \pi^2 Zen\left(1 + \cos^2\theta_{kB}\right)\left|\frac{dB}{dr}\right|^{-1}, \qquad (3)$$

where all quantities are taken at the corresponding point of the resonant surface (r_{res}, θ), n is the resonant particle number density, and θ_{kB} is the angle between the photon wavevector and the magnetic field. For dipolar magnetic field $|dB/dr| \simeq (3/2)B/r$. The current density induced by the twist of the magnetic field line is

$$j_{twist} = \frac{c}{4\pi}\nabla\times\boldsymbol{B} \sim \frac{c}{4\pi}\frac{\delta B}{\Delta x} \sim c\frac{1}{2}\frac{\xi}{\Delta x}\frac{B}{\pi s}, \qquad (4)$$

where $\delta B \sim 2(\xi/s)B$ is the variation of the magnetic field due to twist of the magnetic field line, Δx is the characteristic scale of the electromagnetic field variation, s is the length of the corresponding magnetic field line, and ξ is the oscillation amplitude. Assuming that the plasma density is proportional to the current density (see eq. [B1] in Appendix B), we get for the optical depth

$$\tau \sim \pi^2\kappa\frac{j_{twist}}{c}\frac{2}{3}\frac{r_{res}}{B} \sim \frac{\pi}{3}\kappa\frac{\xi}{\Delta x}\frac{r_{res}}{s}, \qquad (5)$$

where κ is the multiplicity of pair plasma. The maximum optical depth is achieved near the equator, where $s/r_{res} \simeq 2.6$ for $r_{res} \sim 10R_{NS}$; for oscillation mode with the harmonic number l, $\Delta x \simeq \pi R_{NS}/l$. Using these, τ can be estimated as

$$\tau \sim \kappa l\frac{\xi}{3R_{NS}}\frac{r_{res}}{s} \sim \kappa\frac{l}{8}\frac{\xi}{R_{NS}}. \qquad (6)$$

Hence, the modulation of the optical depth of the order of 0.01κ would require oscillation amplitude for the harmonic number $l \simeq 10$ of the order of 1%. Such oscillation would produce shear strain of the NS crust of the order of 10%, implying that the nearest neighbor distance oscillates by about 1% (in Appendix B we argue that κ may be of the order of a few or larger, which would provide optical depth modulation of the order of $\sim$10%). The question is whether such a crustal strain could accommodate the observed QPO in the flux. We now do a more quantitative calculation relating optical depth to crustal strain and flux modulation.

3. AXISYMMETRIC FORCE-FREE MAGNETOSPHERE OF AN OSCILLATING NEUTRON STAR WITH DIPOLE MAGNETIC FIELD

Consider an idealized case of a nonrotating star with dipole magnetic field oscillating in toroidal oscillation mode with small amplitude. We consider disturbance of the magnetosphere by toroidal modes with even values of l and $m = 0$. We use a spherical coordinate system centered at the NS center, with the z-axis parallel to the dipole moment and the same representation for magnetic field and currents in the force-free magnetosphere as in Timokhin (2006). For an axisymmetric force-free field, the toroidal component induced by the oscillations is

$$B_\phi = \frac{4\pi}{c}\frac{I}{r\sin\theta}, \qquad (7)$$

where $2\pi I$ is the total induced poloidal current flowing between the pole and the colatitude θ. For small perturbation of the magnetic field we have

$$\frac{r\sin\theta\, d\phi}{ds} = \frac{B_\phi}{B_{pol}}, \qquad (8)$$

where B_{pol} is the poloidal component of the magnetic field and ds is the distance along the field line. The total twist of the magnetic field line (the difference $\phi_1 - \phi_2$ of the field line footpoint azimuthal angles) $\delta\phi_*$ is

$$\delta\phi_* = \int_{\theta_*}^{\pi-\theta_*}\frac{d\phi}{d\theta}d\theta, \qquad (9)$$

where θ_* is the colatitude of the magnetic field line footpoint in the upper hemisphere. The derivative $d\phi/d\theta$ can be expressed through the derivative $d\phi/ds$ as

$$\frac{d\phi}{d\theta} = \frac{d\phi}{ds}\frac{ds}{d\theta} = \frac{d\phi}{ds}R_{NS}\frac{\sin\theta}{\sin^2\theta_*}\sqrt{1+3\cos^2\theta}, \qquad (10)$$

and $d\phi/ds$ can be obtained from equation (8). Expressing B_ϕ through the poloidal current I (eq. [7]) and performing the integration, we get for the total induced poloidal current

$$I = \frac{1}{2}\delta\phi_*\frac{c}{8\pi}B_0 R_{NS}\frac{\sin^4\theta_*}{\cos\theta_*}. \qquad (11)$$

The total twist of the magnetic field line $\delta\phi_*$ is

$$\delta\phi_*(\theta_*) = 2\frac{\xi(\theta_*)}{R_{NS}\sin\theta_*}, \qquad (12)$$

where $\xi(\theta_*)$ is the displacement of the magnetic field line footpoint. For the toroidal oscillation modes under consideration, the displacement of the NS surface $\xi(\theta_*)$ is

$$\xi(\theta_*) = -\xi_0\frac{dY_{l0}(\theta_*)}{d\theta_*}, \qquad (13)$$

where ξ_0 is the mode amplitude and Y_{l0} is the spherical harmonic of order $l, 0$. The current density can be expressed through the magnetic flux function ψ as

$$j = \frac{\nabla I \times \boldsymbol{e}_\phi}{r \sin\theta} = \frac{dI}{d\psi}\boldsymbol{B}_{\rm pol}. \quad (14)$$

The magnetic flux function ψ for the dipole field is

$$\psi = \frac{B_0 R_{\rm NS}^3}{2}\frac{\sin^2\theta}{r}. \quad (15)$$

The derivative $dI/d\psi$ is

$$\frac{dI}{d\psi} = \frac{dI}{d\theta_*}\frac{d\theta_*}{d\psi} = \frac{dI}{d\theta_*}\frac{1}{B_0 R_{\rm NS}^2 \sin\theta_* \cos\theta_*}. \quad (16)$$

Combining equations (11)–(16) we obtain

$$j = j_0\left(\frac{r}{R_{\rm NS}}\right)^{-3}\sqrt{1+3\cos^2\theta}A_l[\theta_*(\theta, r)], \quad (17)$$

where

$$j_0 = \frac{c}{16\pi}\frac{B_0}{R_{\rm NS}}\frac{\xi_0}{R_{\rm NS}}, \quad (18)$$

$$A_l = \frac{\sin\theta_*}{\cos^2\theta_*}\left(\frac{1+2\cos^2\theta_*}{\cos\theta_*}\frac{dY_{l0}}{d\theta_*} + \sin\theta_*\frac{d^2Y_{l0}}{d\theta_*^2}\right). \quad (19)$$

As an example, we plot in Figure 1 the current density distribution for toroidal mode $l = 10$. The current density is strongly modulated in the latitudinal direction, and therefore, the resonant Compton optical depth is highly anisotropic.

Now we can obtain an expression for the optical depth. Substituting the expression for the current (eq. [17]) into the formula for the optical depth (eq. [3]), we get

$$\tau = \kappa\frac{\pi}{24}\frac{\xi_0}{R_{\rm NS}}r_{\rm res}\frac{1+7\cos^2\theta}{1+3\cos^2\theta}A_l[\theta_*(\theta, r)], \quad (20)$$

Here, for simplicity, as in Thompson et al. (2002) we assume radially streaming photons.

If the resonant surface is located not very far from the NS, the optical depth has several maxima at different colatitudes. The position of these maxima as well as their amplitude will depend on the shape and position of the resonant surface. The latter depends on the magnetic field strength as well as on the energy of the upscattered photons. In Figure 1 we plot three resonant surfaces for different energies of the upscattered photons for the polar magnetic field $B_0 = 10^{14}$ G. It is evident that for different photon energies the angular distribution of the optical depth will be different. In Figure 2 we plot the angular distribution of the optical depth for different energies of the upscattered photons, different oscillation modes, and different magnetic field strengths. In each of these plots we assumed the amplitude of the oscillation mode $\xi_0 = 0.01R_{\rm NS}$. Latitudinal distribution of the oscillation amplitude $\xi(\theta_*)$ as well as the largest component of the strain tensor $e_{\theta\phi}$ (see, e.g., Kosevich et al. 1986) for the considered modes $l = 4$, 8, and 10 are shown in Figures 3 and 4, respectively.

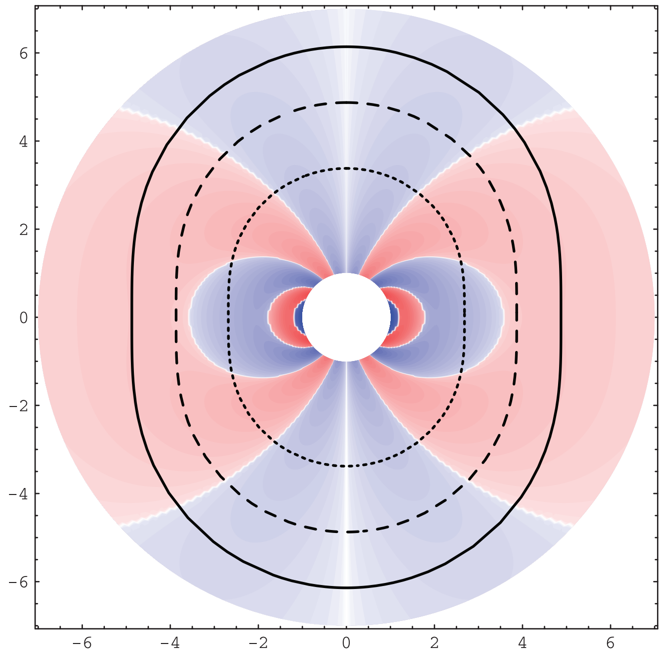

Fig. 1.—Current density distribution in the magnetosphere of a NS with dipole magnetic field oscillating in toroidal mode ($l = 10, m = 0$). The intensity of color in each point is proportional to $|j(r,\theta)|^{1/6}$ (this moderates contrast of the plot). Positive values of the current density are shown by red colors, and negative ones are shown by the blue colors. Black lines show the resonant surfaces, eq. (2), for three photon energies $E_{\rm ph} = 5$, 10, 30 keV, solid, dashed, and dotted lines, respectively. The magnetic field at the pole is assumed to be $B_0 = 10^{14}$ G.

The optical depth distribution has the following properties.

1. The latitude of peak optical depth is strongly energy dependent for the low-latitude peaks and less so for the peaks near the polar axis (see Fig. 1).
2. The maximum optical depth does not depend significantly on the initial photon energy (Fig. 2, *left*).
3. For moderate l ($l \lesssim 10$), the optical depth produced by mode amplitude ξ_0 varies little with l, although the angular dependence is naturally different (see Fig. 2, *middle*). For larger l, equation (6) gives a rather good estimate for the maximum value of the optical depth, which, for the same mode amplitude at the magnetic field line footpoint $\xi(\theta_*)$, grows with increasing l.
4. The maximum value of the optical depth does not depend significantly on the strength of the magnetar magnetic field for commonly accepted parameters of $B_0 = 10^{14}$–10^{15} G (see Fig. 2, *right*).

We consider now three possibilities for modulation of hard X-radiation: (1) modulation is independent on photon energy change and depends only on anisotropic obscuration of photons generated within the resonant surface; and (2) modulation is achieved by upscattering of photons in energy by either (*a*) nonrelativistic electrons or (*b*) at least moderately relativistic electrons.

Case 1 requires an anisotropic optical depth of the order of 10%. This is easily achieved with the multiplicity κ of the order of 10, which may be reached because pair production requires primary electron energies of the order of $\simeq 10mc^2$, where m is the electron mass (see Appendix B).

For case 2*a* the fluctuation of the photon flux is increased by the Compton-Getting effect as

$$\frac{\Delta F_{\rm ph}}{F_{\rm ph}} = \tau\frac{U}{c}\left[-\frac{d\log F_{\rm ph}(E_{\rm ph})}{d\log E_{\rm ph}}\right] \approx 4\tau\frac{U}{c}, \quad (21)$$

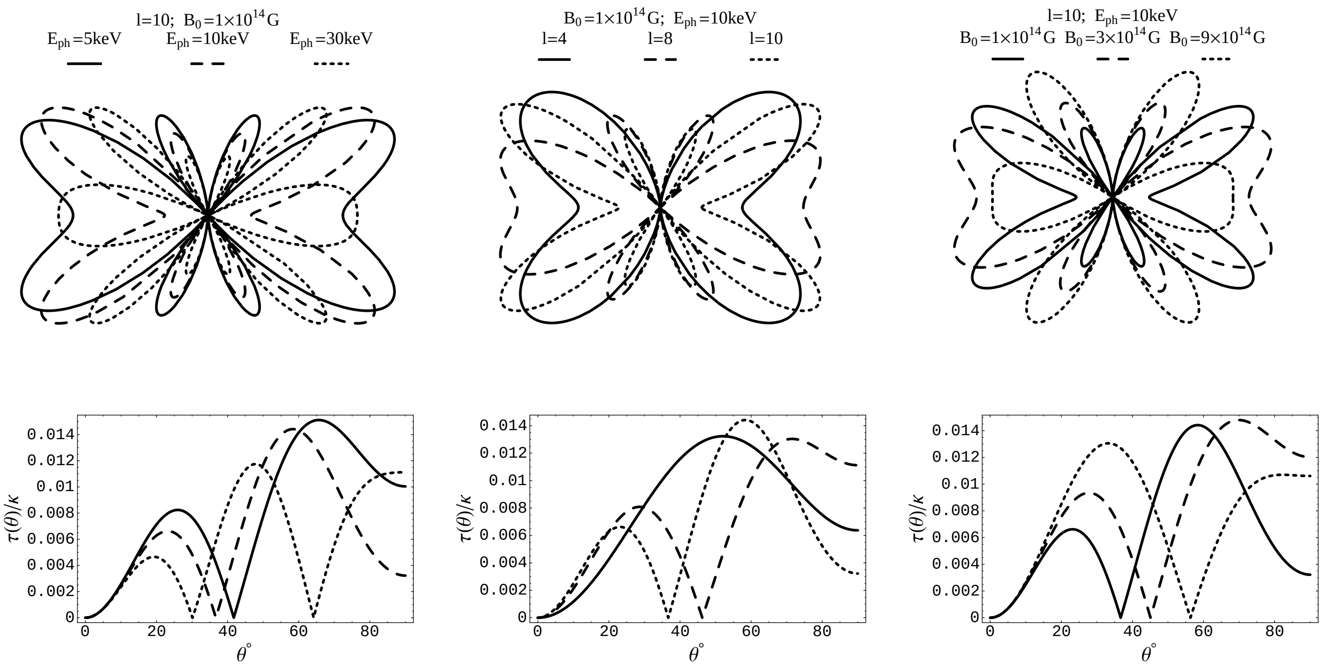

FIG. 2.—Angular distribution of the optical depth to resonant Compton scattering for radially moving photons in the magnetosphere of an oscillating NS for an amplitude of $\xi_0 = 0.01R_{\rm NS}$. *Top*: Angular distribution $\tau(\theta)/\kappa$ shown in polar coordinates $[\tau(\theta)/\kappa, \theta]$. *Bottom*: Same distribution is shown as a plot $\tau(\theta)$ for $\theta \in [0, 90°]$. *Left*: Optical depth for three different initial energies of the scattered photons ($E_{\rm ph} = 5$, 10, and 30 keV); the magnetic field $B_0 = 10^{14}$ G and the harmonic number of the toroidal mode $l = 10$ are fixed. *Middle*: Optical depth for three different modes ($l = 4$, 6, and 10); the magnetic field $B_0 = 10^{14}$ G and the initial energy of the scattered photons $E_{\rm ph} = 10$ keV are fixed. *Right*: Optical depth for three different values of the NS magnetic field ($B_0 = 1 \times 10^{14}$, 3×10^{14}, and 9×10^{14} G); the harmonic number of the toroidal mode $l = 10$ and the initial energy of the scattered photons $E_{\rm ph} = 10$ keV are fixed.

where U is the characteristic electron velocity at the scattering point (the sum of bulk, thermal, and drift velocity) and $F_{\rm ph}$ is the number density of photons with the energy $E_{\rm ph}$. We have used the time-integrated photon spectrum $F_{\rm ph}(E_{\rm ph})$ of the tail phase reported by Hurley et al. (2005). This is more than the contribution of case 1 if $U/c > 1/4$. It gives at least 6% modulation of the hard X-ray flux for displacement amplitude of the order of $0.01R_{\rm NS}$.

Case 2*b* minimizes the optical depth implied by the twisted field, but on the other hand, it enables the scattering process to lift photons directly from the thermal peak at $E_{\rm ph} \simeq 10$ keV to the high-energy tail. We estimate that the unmodulated tail represents about 0.1 of the total photon number (see Hurley et al. 2005), so a QPO modulation amplitude in the high-energy tail of 10% would require an optical depth to resonant inverse Compton scattering of about 1%.

For oscillation modes with $l \leq 10$, the maximum value of τ/κ reaches ~1% (1.5%) when the displacement of the NS crust is 0.6% (1%) (see Fig. 3). The strain of the crust caused by the modes $l \leq 10$ is typically several percent and never exceeds 10% (see Fig. 4). By terrestrial standards, strain of even several percent is large enough to raise questions about the proposed mechanism. One possibility is that large pair multiplicity, κ, lessens the requirements on the displacement to achieve the necessary perturbation in the optical depth, τ. Another possibility is that the strong magnetic field may provide a rubbery consistency to the crust material so that it could withstand larger reversible strain than brittle crystal. (This would be an implication of the likely possibility that many other modes were excited in addition to those that produced observable QPO behavior.) It should also be recalled that elastic limits in terrestrial materials are generally established

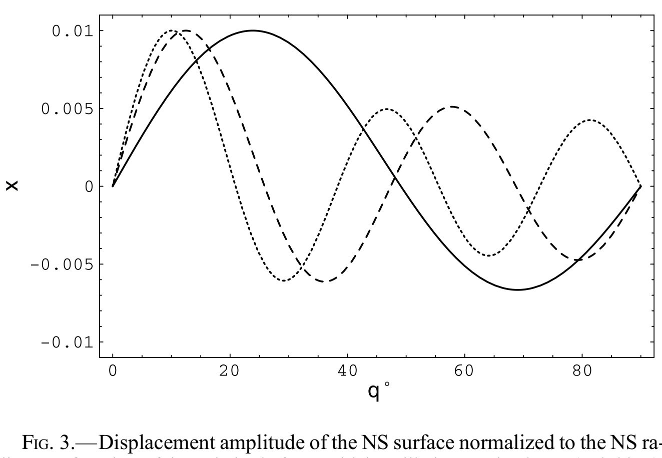

FIG. 3.—Displacement amplitude of the NS surface normalized to the NS radius as a function of the colatitude for toroidal oscillation modes $l = 4$ (*solid line*), 8 (*dashed line*), and 10 (*dotted line*). The mode amplitude $\xi_0 = 0.01R_{\rm NS}$.

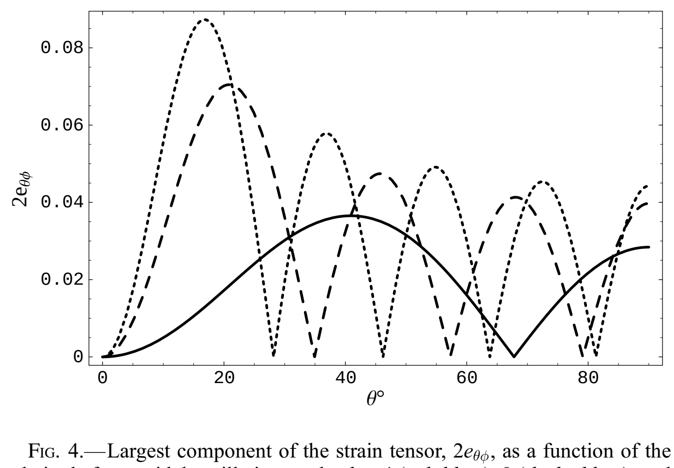

FIG. 4.—Largest component of the strain tensor, $2e_{\theta\phi}$, as a function of the colatitude for toroidal oscillation modes $l = 4$ (*solid line*), 8 (*dashed line*), and 10 (*dotted line*). The mode amplitude $\xi_0 = 0.01R_{\rm NS}$.

by the surface effects, microfractures, impurities, dislocations, and other imperfections. One should be cautious in generalizing these limits to a magnetar's crust, which is likely to be pure, nearly perfect, edge-free (in the context of torsional oscillations), and strongly magnetized.

Angular modulation of the optical depth to the resonant Compton scattering may result in temporal modulation of the hard X-ray photons. Indeed, the QPO features are observed only at specific phases of the X-ray light curve (Strohmayer & Watts 2006; Israel et al. 2005). A quantitative fit to the light curves is beyond the scope of this article, but we suspect that the angular dependence of the optical depth is the main reason that the QPO features are rotational phase dependent. Altogether, the proposed scenario seems to be in good agreement with the interpretation of the largest amplitude ($\sim$10%) QPOs as somehow resulting from NS oscillations with harmonic number $l \lesssim 10$ (e.g., Strohmayer & Watts 2006; Israel et al. 2005).

Clearly, an optical depth in the corotating magnetosphere of order 10% along some directions and not others creates periodic luminosity variations of order 10% at about the oscillation frequency of the optical depth variation, as it is much larger than the rotation frequency of the magnetar. Rotational sidebands, although predicted to be present in principle, are probably unresolvable given the width of the line.

A detailed radiative transfer calculation for either elastic scattering and inelastic scattering is beyond the scope of this paper and, in any case, limited by our ignorance of the scatterer velocity distribution and magnetic field geometry.

4. DISCUSSION

In all of the above, we have assumed for simplicity that the torsional oscillations and magnetic field are coaxial and axisymmetric. In reality, a more complex topology of the magnetic field is likely, hence a more complicated l dependence. The general principle nevertheless remains that some modes twist field lines more than others and that some field lines will be twisted more than others for any given mode.

The fact that QPOs are seen at high photon energies but not at low photon energies favors upscattering over anisotropic obscuration. Our interpretation of the QPO peaks in the tail-phase emission of the giant flares seems consistent with the overall picture of magnetar emission that has emerged over the past several years. In particular, it is consistent with the view that the hard X-radiation is due to nonthermal reprocessing by the magnetospheric current carriers or direct emission from a hot corona. It is also consistent with the interpretation that these oscillations are torsional rather than some other kind.

One unsurprising implication of the remarkably large amplitude of the QPO modulation is that the crust receives an enormous amount of mechanical energy from the giant flare. We estimate over 10^{42} ergs per mode if the amplitude of oscillation is 10^{-2} NS radii and the QPO frequency centered on 150 Hz. Assuming that the modes are indeed torsional modes, it follows that their mechanical energy deposition extends throughout the entire crust. The fact that the measured Q of these oscillations is always much less than the number of periods that the QPO behavior lasts is quite curious, and the most obvious interpretations are either that (1) the oscillations damp on a short time compared to the overall duration, requiring continuous excitation; or (2) many modes are excited ($\gg 10$, as at least 10 separate frequencies are detected in the individual source) with an unresolved spread in frequencies clustering around the observed central ones. Either interpretation implies that far more than 10^{43} ergs are excited. We may suppose that over 10^{45} ergs are released in the inner crust as mechanical motion. This is far more than the emission during the tail phase and probably goes mostly into heating the crust. This is consistent with the assumption that over 10^{45} ergs can be deposited in the deep crust that was made (Kouveliotou et al. 2003; Eichler et al. 2006) to account for observed long-term X-ray afterglow from magnetars 1627−41 and 1900+14.

In each case, a decent fit to the observed cooling curve requires a large enough mass of the NS that the direct Urca process cools the core (at least 1.4 $M_\odot$). Combining this result with analysis shown in Figure 5 in Strohmayer & Watts (2005) could set interesting limits on the equation of state of the NS.

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APPENDIX A

GOLDREICH-JULIAN CHARGE DENSITY

The twist of magnetic field lines induces currents flowing along the field lines. In addition, motion of the field lines induces an electric field, and in order to keep the magnetosphere force-free, the charge density in the magnetosphere must contain an oscillating component—the "oscillational" Goldreich-Julian (GJ) charge density. Let us compare minimal particle number densities required by changes in the GJ charge density and by current supporting twist of the magnetic field lines.

The oscillational GJ charge density could be estimated as

$$\rho_{\rm GJ} = \frac{1}{4\pi}\nabla\cdot\boldsymbol{E} \sim \frac{1}{4\pi}\frac{v}{c}\frac{B}{\Delta x} \sim \frac{1}{2}\frac{\xi}{\Delta x}\frac{B}{\lambda_{\rm QPO}}, \tag{A1}$$

where Δx is the characteristic scale of the electromagnetic field variation, v is the amplitude of the oscillational velocity, and ξ is the oscillation amplitude. The current density induced by the twist of the magnetic field line is given by equation (4). The ratio of the corresponding particle number densities is thus

$$\frac{n_{\rm twist}}{n_{\rm GJ}} \equiv \frac{j_{\rm twist}}{c}\frac{1}{\rho_{\rm GJ}} \sim \frac{\lambda_{\rm QPO}}{\pi s} > 1; \tag{A2}$$

the last inequality follows from estimates of the sizes of the resonant surface and the wave zone given in § 2. We note that this estimate is valid for any point in the magnetosphere and also for a point at the resonant surface. The quantity s is then the length of the magnetic field line from the considered point to the corresponding symmetrical point at the resonant surface. Taking into account that the resonant surface is at distances of the order of $\sim 4R_{NS}$–$10R_{NS}$, we conclude that the particle number density modulation due to changes in the GJ charge density is always less than the modulation caused by twist of the magnetic field lines, even for oscillation with frequencies of the order of 1 kHz, when the wave zone size is $30R_{NS}$.

The magnetic field lines get twisted only if their footpoints are moving in opposite directions. Hence, in an axisymmetric background magnetic field only half the oscillation modes twist the field lines. The current induced by the twist flows along the field line and changes the particle density in proportion to the local value of the magnetic field, so the current density with the distance from the NS would decrease as $\sim r^{-3}$. These are the modes, we propose, which produce observable quasi-periodic modifications in the hard X-radiation in the tail of the giant flares.

The modes with different symmetry, when the field line footpoints move in the same directions, produce only modulation of the particle number density due to changes in the GJ charge density. Such a situation was considered in Timokhin et al. (2000) and Timokhin (2007), where an analytical three-dimensional model of the force-free magnetosphere of an oscillating NS was constructed under the assumption that the current along magnetic field lines induced by the stellar oscillations is negligibly small. In that case, only the *charge* density near the NS is modulated, which as we have shown above introduces much smaller modulation to the particle number density compared to the modes twisting magnetic field lines with the same oscillation amplitude. Moreover, far from the star, near the resonant surface, the alteration of the particle number density is very weak because ρ_{GJ} falls very rapidly with the distance from the NS, as $r^{-(2l+1)}$ for a mode with harmonic number l. The changes in the particle number density near the resonant surface, in that case, are negligibly small, and such modes will not produce detectable modulation of the hard X-ray emission.

APPENDIX B

PLASMA PRODUCTION IN THE MAGNETOSPHERE

At the tail phase of the giant burst, a magnetically trapped fireball radiates the energy stored during the initial outburst. Luminosity at this stage, $\sim 10^{42}$ ergs s^{-1}, is highly super-Eddington. More exactly, the fireball emits radiation in the extraordinary mode, for which radiation opacities are strongly suppressed by magnetic field, and the fireball luminosity is in fact the Eddington luminosity for this radiation (Thompson & Duncan 1995). However, extraordinary photons with energy $E_{ph} > 40$ keV are split in the magnetosphere to ordinary mode photons, for which the magnetized opacities are comparable with the nonmagnetized ones. Therefore, any plasma in the magnetosphere will be pushed upward by the radiation from the fireball. However, the magnetosphere could not become empty because it should support electric currents generated during the initial outburst when the magnetosphere was not force-free. If the plasma density falls below the value necessary to maintain the current, the electric field (displacement current) arises and accelerates particles until they produce electron-positron pairs (Thompson et al. 2002; Beloborodov & Thompson 2007).[3]

During the tail phase, the pairs are easily produced via γ-γ conversion of photons upscattered off relativistic electrons. The photon density in the magnetosphere is $n_{ph} = 10^{25} L_{41}/\varepsilon_{50}$ cm^{-3}, where $L = 10^{41} L_{41}$ ergs s^{-1} is the luminosity at the photon energy $\varepsilon = 50\varepsilon_{50}$ keV. The scattering cross section of the ordinary photon is $\sigma = \sigma_T \sin^2\theta'$, where θ' is the angle between the photon and the magnetic field in the proper electron frame. So the electron with the Lorentz factor $\gamma = 10/\varepsilon_{50}$ produces a photon with the energy $\gamma^2\varepsilon = 10mc^2/\varepsilon_{50}$ after passing a length $l = (\sigma n)^{-1} = 20\varepsilon_{50}/L_{41}$ cm. This photon is easily converted into a pair by collision with a background photon. In principle, electrons with lesser Lorentz factors could also produce pairs by upscattering photons with higher energies, however, the density of photons sharply decreases with the energy; therefore, one can expect that the electron is accelerated to at least $\gamma > 10$ before it produces a pair. After the first pairs are produced, they screen the electric field and acceleration ceases. The accelerated electrons rapidly lose their energy upscattering photons. As any ordinary photon with energy >1 MeV is converted into a pair when it acquires an appropriate angle with the magnetic field, one can expect that eventually any primary electron produces at least a few pairs, the number of pairs being proportional to the energy the electron acquires within the gap. An important point is that the number density of the primary electrons is proportional to the current density, $n_{primary} = j/ec$, because there are no other particles in the gap. Therefore, one can expect that the total number of pairs is proportional to the current

$$n = \kappa j/ec, \tag{B1}$$

where the multiplicity κ is at least of the order of a few but may be larger. In order to find the multiplicity, one should solve self-consistently the electrodynamics of the gap together with the particle dynamics and pair production (Beloborodov & Thompson 2007), which is beyond the scope of this paper.

[3] This mechanism for pair production, which is linear in the twist of the field lines, is somewhat different from that invoked by Thompson & Duncan (2001), who invoked nonlinear Alfvén wave disturbances to explain the constant tail emission via a trapped pair fireball. Whether it can replace their mechanism or merely supplement it as a mechanism for primary emission is not the point and is beyond the scope of this paper, which is more concerned with the reprocessing of the primary tail emission at the surface of cyclotron resonance.

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